$\therefore 1521=13^2+34^2+14^2=13^2+26^2+26^2=34^2+2^2+19^2=26^2+22^2+19^2$ $=26^2+2^2+29^2=14^2+22^2+29^2=14^2+35^2+10^2$.

III. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.

Let $1521=m^2$, and x^2, y^2, and z^2 represent the three squares; then $x^2+y^2+z^2=m^2$, and $x^2=m^2-(y^2+z^2)=m^2-2pm+p^2$. $2pm=p^2+y^2+z^2$. Let $y=tp$ and $z=sp$, then $2m=p(s^2+t^2+1)$. Restoring the value of m, $p^2=78/(s^2+t^2+1)$, in which s and t may be any rational numbers. Take $s=1$, $t=1$, then $p=26$; $x=m-p=13$, $y=tp=26$ and $z=sp=26$, and $13^2+26^2+26^2=1521$. Take $s=2$, and $t=1$, $p=13$; $x=26$, $y=13$, $z=26$. Take $s=3$, $t=4$, then $p=3$; $x=36$, $y=12$, $z=9$. Take $s=2$, $t=3$, $p=\frac{39}{7}$; $x=\frac{234}{7}$, $y=\frac{78}{7}$, $z=\frac{117}{7}$, and $[(234)^2+(78)^2+(117)^2]/49=1521$.

While this is a solution of the question read *literally*, of course, I understand that the proposer intends to call for integral numbers; but I have obtained seven integral results, only *by trial.*

Also solved by *SYLVESTER ROBINS*, and *G. B. M. ZERR.*

66. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find two cubic proper fractions whose product is a square proper fraction. Can a *general* solution be made?

Solution by E. L. SHERWOOD, A. M., Superintendent City Schools, West Point, Miss.; CHARLES CARROLL CROSS, Libertytown, Md.; and G. B. M. ZERR, A. M., Ph. D., Chester High School, Chester, Pa.

Let a^6/b^6, c^6/d^6 be the fractions, $a<b$, $c<d$.

Then $a^6c^6/b^6d^6=(a^3c^3/b^3d^3)^2$.

Let $a=1$, $b=2$, $c=2$, $d=3$.

$\therefore a^6/b^6=\frac{1}{64}$, $c^6/d^6=\frac{64}{729}$;

$\therefore a^6/b^6=(\frac{1}{4})^3$, $c^6/d^6=(\frac{4}{9})^3$; $a^6c^6/b^6d^6=(\frac{1}{27})^2$.

Other fractions can easily be found.

Also solved by *J. H. DRUMMOND.*

67. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find (1) four consecutive numbers whose sum is a square, and (2) four consecutive numbers the sum of whose squares is a square.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let $x-1$, x, $x+1$ and $x+2$=four consecutive *integers.*

(1) Then their sum$=4x+2=2(2x+1)=2$ *times an odd number.* But this result can never be a square. $\therefore$ The sum of four consecutive integers can *not* be a square.

(2) The sum of their squares$=4x^2+4x+6=2[2(x^2+x+1)+1]=2$ *times an odd number*, which result can never be a square. $\therefore$ The sum of the squares of four consecutive integers can *not* be a square.

If, however, four consecutive numbers may be considered as four *fractions* whose denominators are the same number and equal to 2 *times a square*, and

whose numerators are consecutive integers, then we are able to fulfill the *first part* of the problem.

Of any four consecutive integers we have shown that their sum is 2 *times an odd number*. Now when this odd number is a *square*, we can find four consecutive fractions whose sum is a square, by making the denominators$=2m^2$ and the respective numerators$=2n(n+1)-1$, $2n(n+1)$, $2n(n+1)+1$, and $2n(n+1)+2$. Whence we have $\{[2n(n+1)-1]/2m^2\}+\{[2n(n+1)]/2m^2\}+\{[2n(n+1)+1]/2m^2\}+\{[2n(n+1)+2]/2m^2\}=(2n+1)^2/m^2$.

When $n=m=1$, we find $\frac{3}{2}+\frac{4}{2}+\frac{5}{2}+\frac{6}{2}=3^2$. When $n=1$ and $m=2$, we have $\frac{3}{8}+\frac{4}{8}+\frac{5}{8}+\frac{6}{8}=(\frac{3}{2})^2$. When $n=2$ and $m=1$, we find $\frac{11}{2}+\frac{12}{2}+\frac{13}{2}+\frac{14}{2}=5^2$, etc.

II. Solution by CHARLES CARROLL CROSS, Libertytown, Md.

(1) Combining the consecutive numbers we find that $1+2+3+4$, $5+6+7+8$, $6+7+8+9$, $7+8+9+0$, and $0+1+2+3$ are the only combinations whose terminal figure produces the terminal figure of a square. The first and third combinations can never produce a square number, because a square number whose terminal figure is 0 is always preceded by 0. The second and last combinations cannot produce a square number, because a square number whose terminal figure is 6 is always preceded by an odd number. The fourth combination can never be a square number, because a square number whose terminal figure is 4 is always preceded by an even number. Hence (1) is incorrect.

(2) In the *Mathematical Visitor*, Vol. I, No. 5, page 156, Dr. Martin has shown that the sum of three, of four, and of five consecutive squares, cannot be a square number. Hence (2) is also incorrect. [See also *Mathematical Magazine*, Vol. II, No. 6, page 92.]

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

(1) Let x, $x+1$, $x+2$, $x+3$ be the numbers.

$\therefore 4x+6=a^2$, or $x=(a^2-6)/4$.

$\therefore (a^2-6)/4$, $(a^2-2)/4$, $(a^2+2)/4$, $(a^2+6)/4$ are the numbers.

(2) $4x^2+12x=b^2-14$.

$\therefore x=\frac{1}{2}\sqrt{(a^2-5)}-\frac{3}{2}$, where (a^2-5) must be a square.

Let $a=3$, then $-\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, are the numbers.

Let $a=2\frac{1}{3}$, then $-\frac{7}{6}$, $-\frac{1}{6}$, $\frac{5}{6}$, $\frac{11}{6}$ are the numbers.

And so for other values of a.

Also solved by *EDWARD R. ROBBINS*, and *J. H. DRUMMOND*.